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and the points F_1, E, F_2, D are concyclic. The perpendicular bisectors of F_1F_2 and DE therefore intersect at C, the required center.

It may be observed that the second construction is a direct generalization of the one (when the asymptotes and vertex are given) found in all elementary text-books on Analytic Geometry. Would it not be well for the latter to contain the above simple constructions?

The University of Chicago, October, 1902.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

161. Proposed by F. M. SHIELDS, Coopwood, Miss.

If 1 man, 1 boy, and 1 girl catch 1 trout, 1 perch, and one minnow in 5 minutes, and 1 man, 2 boys, and 3 girls catch 1 trout, 2 perch, and 3 minnows in 6 minutes, how many minutes will be required for 2 men, 3 boys, and 4 girls to catch 5 trout, 11 perch, and 17 minnows?

Remarks by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

This problem is indeterminate. Let A_1, A_2, and A_3 be the activities of a man, boy, and girl, respectively; and E_1, E_2, and E_3 the work required to catch a trout, a perch, and a minnow, respectively. Then

$$5(A_1+A_2+A_3)=E_1+E_2+E_3\text{....(1), and}$$
$$6(A_1+2A_2+3A_3)=E_1+2E_2+3E_3\text{....(2).}$$

From these two conditions it is required to find t so that the following condition is satisfied, viz:

$$t(2A_1+3A_2+4A_3)=5E_1+11E_2+17E_3.$$

This is clearly impossible in general. By assuming certain other conditions as, for example, the ratios of the activities, and the ratios of the work, a solution may be effected.

162. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

A trolly road is built between two towns, and it is found that the gross annual receipts amount to 20% of the original cost; the annual cost of repairs 2% of the original cost; and the working expense is $3000 in addition to 20% of the net receipts. After a year a second road is built at the same cost as the first and it is found that the gross receipts and working expenses per year are doubled, while the cost of repairs for the new road is 1% of cost. If the receipts for both roads is \$72,500, find the cost of each road, and the net receipts the first year.

Solution by the PROPOSER.

Let 100%=cost of each road, 20%=gross receipts first year, 2%=repairs first year.

Working expenses=\$3000+20% of (20%−2%−working expenses).

∴ $\frac{6}{5}$ working expenses=\$3000+3.6%.

∴ working expenses=\$2500+3%.

The second year, gross receipts=40%, repairs=3%, working expenses=\$5000+6%.

∴ \$72,500=40%−3%−6%−\$5000.

∴ 31%=\$77,500, 100%=\$250,000, cost of each road.

Net receipts first year=20%−2%—\$2500−3%=15%—\$2500=\$35,000.

Also solved by *W. A. CLEMENSER*, Heidelberg University, Tiffin, O.; and *C. A. SHORT*, Assistant Professor of Mathematics, Delaware College, Delaware.

ALGEBRA.

147. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

Prove that $x=a^x$ has never more than two real roots, and find the condition for no real roots.

Solution by the PROPOSER.

$x=a^x$. ∴ $a=x^{1/x}$. $a_1=\frac{x^{1/x}}{x^2}(1-\log x)$ where $a_1=\frac{da}{dx}$ and $a_1=0$ for $x=0, e, \infty$. For $x>1$, a is >1; for $0<x<1$, $0<a<1$, for $x=1$, $a_1=1$.

∴ The curve represented by the equation touches axis of x at $x=0$, and its ordinates increase from 0 to 1 with its abscissæ; at $x=1$ it is touching the bisector of the axes. From 1 to e for x, ordinates increase from 1 to $e^{1/e}$, and as x increases from e to ∞, the ordinates decrease from $e^{1/e}$ to 1.

∴ For $0<a<1$, a real root between 0 and 1,

$1<a<e^{1/e}$, a real root between 1 and $e^{1/e}$, or another between e and ∞.

$a=e^{1/e}$, the root is e.

If $a>1$ and $re^{\theta i}$ an imaginary root, then if $ra.\theta=u$, $r\sin\theta=v$ we have $re^{\theta i}=e^{\log a re^{\theta i}}=e^{\log(a+\theta i)}$ and the equation becomes, on equating real and imaginary parts, $\log r=u\log a$; $\theta=v\log a$. The intersection of these curves gives the imaginary roots required, but we must reject roots between the lines $v\log a=(2k+1)\pi$ and $v\log a=(2k+2)\pi$. If $a<1=1/b$ the curves are $r=b^{-u}$, $\theta=-v\log b$, and these curves are the images of the two previous, and the branches giving the roots of the equation proposed are the images of the branches which we previously rejected, and vice versa.

For a full discussion of this equation see Nouville's *Annales*, 1896, p. 548; 1897, p. 54.